

Darcy Flow at Fluid Pressure Equal Rock Pressure

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Darcy's flow equation states: $J = -K (dh/dx)$ where J represents fluid volume flux ($\text{cm}^3 \text{cm}^{-2} \text{s}^{-1}$) and K denotes hydraulic conductivity in a representative volume (cm s^{-1}). h , the hydraulic head, is a measure of fluid energy cast in units of length (cm) in the flow direction x (cm). For fluid pressure, P_F , independent of rock pressure, P_R , fluid energy is from (1) position in the gravitational field, (2) fluid pressure and (3) motion (trivial). For P_F fixed by P_R , the position in the gravitational field and P_F are not independent. Energy of the fluid depends only on position in the gravitational field (with energy of motion trivial). Tectonic stresses in the solid rock are not important as an energy source as they operate over much longer time frames and the fluid itself is in hydrostatic equilibrium. Tectonic stresses become important in how they effect K .

Fluid is transported due to a density difference between rock and fluid of $\rho_r - \rho_f = \Delta\rho$. The loss in gravitational energy due to fluid transport towards the earth's surface is what drives fluid flow. Dimensionally one can consider this mechanical energy drive of the fluid per unit volume.

$$E_v = (ML^2/T^2)/(L^3) = (L)(L/T^2)(M/L^3).$$

With elevation, z (L), acceleration of gravity, g (L/T^2) and $\Delta\rho$ (M/L^3), E_v equals $E_v = zg\Delta\rho$ If the equation is divided by $\Delta\rho$ the result is energy per unit mass,

$$E_m = E_v/\Delta\rho = zg$$

Dividing by g one has energy per unit weight, E_w , or energy calibrated in terms of length, that is elevation, z : $E_w = E_v/(g\Delta\rho) = z$ Therefore, Darcy's law at rock pressure can be written as

$$J = -K' g \Delta\rho (dz/dx)$$

$$K \text{ then equals } K = K' g \Delta\rho$$

In the middle to lower crust: K' for a representative volume varies greatly. It is not clear that a representative volume even exists at mid-crustal levels. That is, the scales of heterogeneity in the crust expand along with the representative volume. No matter what the size of the volume considered there exists larger fractures that are not considered by the continuum model. Also, with $P_F = P_R$, K' depends on the flow rate. Therefore, it is better to construct an explicit model of flow in a volume. Flow will be along grain boundaries in some cases and in large through-going fractures in others. In any case this

flow can be modelled as flow between 2 parallel plates with some tortuosity, τ . τ accounting for the orientation of the fractures relative to the flow path. For single fracture flow it is 1 while for grain boundary flow it is between 0.5 and 0.8 (Bear, 1975).

For laminar flow between 2 parallel plates K' is given by:

$$K' = d^3 l \tau / (12)$$

where d is the width of the fracture (cm), l represents the length of fractures (cm) per unit cross sectional area (cm^2) of the volume in the flow direction and τ denotes viscosity of the fluid (poise). Darcy's law for fluid flow at $P_F = P_R$ can then be written as:

$$J = (d^3 l \tau g \Delta\rho / 12)(dz/dx)$$

With flow at an angle θ from the vertical one has

$$(dz/dx) = \cos\theta, \text{ so that}$$

$$J = (d^3 l \tau g \Delta\rho \cos\theta) / 12$$

This is Darcy's law for fluid flow at $P_F = P_R$ where flow is modelled as laminar fracture flow with some tortuosity (Walther & Wood, 1984; Walther, 1994). As an example consider flow along 1 mm grain boundaries at 45° to the vertical with $\Delta\rho = 1.9 \text{ g cm}^{-3}$ and $\tau = 0.7$. Using an average regional metamorphic flow rate, $J = 5 \times 10^{-10} \text{ cm}^3 \text{ per cm}^2 \text{ s}^{-1}$ and $\tau = 1.5 \times 10^{-3}$ poise (Walther and Orville, 1982) gives $d = 790 \text{ \AA}$. This width is above the $\sim 200 \text{ \AA}$ width at which grain boundary fractures seal as calculated from fluid inclusion planes (Walther and Orville, 1982). The velocity, v , is 0.7 yr^{-1} . If the flow is at 86° to the vertical, the flow would be ~ 10 times slower. However, flow rates increase substantially as fracture spacing increases. Using the parameters above for flow accommodated in 100 m of fracture per 100 m^2 , $v = 130 \text{ m yr}^{-1}$.

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